Adequate sets of connectives for LTL

X is completely orthogonal to the other connectives

- X does not help in defining any of the other connectives.
- The other way is neither possible

Each of the sets {U,X}, {R,x}, {W,X} is adequate

- {U,X}
  - φ R ψ ≡ ¬ (¬ φ U ¬ ψ)
  - φ W ψ ≡ ψ R (φ Ç ψ) ≡ ¬ (¬ ψ U ¬(φ Ç ψ))
- {R,X}
  - φ U ψ ≡ ¬ (¬ φ R ¬ ψ)
  - φ W ψ ≡ ψ R (φ Ç ψ)
- {W,X}
  - φ U ψ ≡ ¬ (¬ φ R ¬ ψ)
  - φ R ψ ≡ ψ W (φ Æ ψ)

Theorem:

U ψ ≡ ¬ (¬ ψ U (¬ φ Æ ¬ ψ)) Æ F ψ

Proof: take any path s Proof: take any path s 0 → s1 → s 2 → in any model 0 → s1 → s 2 → … in any model

- Suppose s 0 ² φ U ψ
  - Let n be the smallest smallest number s.t. sn ² ψ
    - We know that such n exists from φ U ψ. Thus, s0 ² F ψ
    - For each k < n, sk ² φ since φ U ψ

  - We need to show s 0 ² ¬(¬ ψ U (¬ φ Æ ¬ ψ))
    - case 1: for all i, si 2 ¬ φ Æ ¬ ψ. Then, s0 ² ¬(¬ ψ U (¬ φ Æ ¬ ψ))
    - case 2: for some i s case 2: for some i, si ² ¬ φ Æ ¬ ψ. Then we need to show Then, we need to show
    - (*)for each i >0, if si ² ¬ φ Æ ¬ ψ, then there is some j < i with sj 2 ¬ ψ (i.e. sj ² ψ)
    - Take any i >0 with si ² ¬ φ Æ ¬ ψ. We know that i > n since s0 ² φ U ψ. So we can take j=n and have sj ² ψ
  - Conversely, suppose s 0 ² ¬(¬ ψ U (¬ φ Æ ¬ ψ)) Æ F ψ
    - Since s0 ² F ψ, we have a minimal n as before s.t. sn ² ψ
      - case 1: for all i, si 2 ¬ φ Æ ¬ ψ (i.e. si ² φ Ç ψ). Then s0 ² φ U ψ
      - case 2: for some i s ² φ Æ ψ We need to prove for any i <i